The Mathematical Model of the Power Transformer Considering the Parasitic Capacitances

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***Abstract*— The mathematical model of a distribution transformer is presented in this paper. Compared to the normally used equivalent circuit diagram of the transformer, this mathematical model considers also the parasitic capacitances of the windings. The components of the extended equivalent circuit were determined by measurement and a mathematical model was created. It turns out that this kind of the mathematical model could be used for the study of transformer transients.**

Keywords—transformer; parasitic capacitnace; winding; magnetizing reactance

I. Introduction

Nowadays, the renewable energy sources are increasingly utilized in the grid. Their operation is mostly controlled by the inverters and their output is sent out through the transformer to the grid. As the output voltage of these inverters is not often ideal sinusoid, it could result in various phenomena in the transformer that need to be investigated. To study these phenomena it is necessary to establish a mathematical model of the particular transformer. An exact equivalent circuit diagram is needed for this purpose. Many authors utilize a simplified equivalent circuit for the modelling of the transformer that takes into account just winding resistances, leakage reactance, magnetizing reactance and resistance representing the effect of the iron losses. For the modelling of the transformers with respect to the higher-order harmonic components, it is necessary to consider the parasitic capacitances of the primary and the secondary winding and the capacitance between the windings. Therefore, the authors decided to create the mathematical model of the transformer described in the paper.

II. Model of Transformer

There is a 22 kV network model in scale 1:100 at our department that is used for the study the influences of the renewable energy sources connection with respect to the quality of the network or control of its operation. This model includes also a model of a distribution transformer 22/0.4 kV represented shown in Fig. 1. Its parameters are U_1 = 220V, U_2 = 4.2 V, S_n = 63 VA and its connection is Dy1. Further parameters from the nameplate of the transformer are given in Table I. This transformer is used for setting up a mathematical model of distribution transformer.

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TABLE I. Transformer Nameplate

TYPE: T3N-0.063-220 / 4.2 Dy1	S = 63 VA
PRI: 3 x 220 V T 0.2 A	**SEC:** 3 x 4.2 / 8.7 A
Rajec, Slovakia EN 61 558 s.n. 1158 / 2015	

As the basic kinds of the measurements necessary to determine the equivalent circuit diagram of the transformer can be considered the no-load and short-circuit measurements. Not less important is the measurement of the winding resistance with direct current. Measurement of the transformer turns ratio is carried out using the no-load measurement [1].

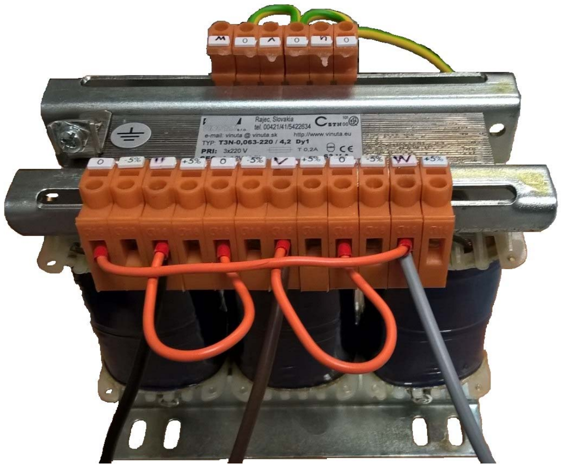

Fig. 1. The used model of the distribution transformer

Firstly, the measurements of the winding resistances using ohm's method was carried out (Fig. 2). Due to measurement accuracy, each measurement was repeated three times and then the mean value was calculated. The primary winding resistance of this transformer is R_1 = 98.76 Ω. Secondary winding resistance is R_2 = 0.035 Ω. The turn ratio measurement was carried out using the no-load measurement and measured values of the voltages are: U_1 = 219.4 V, U_2 = 2.48 V. These values were used to determine the turns ratio:

$$p = \frac{U_1}{U_2} = \frac{219.4}{2.48} = 88.47\,, \qquad (1)$$

where the primary voltage U1 presents line-to-line voltage due to Dy1 connection and the secondary voltage is phase voltage.

It is because the ratio is determined by the number of turns of the related phase.

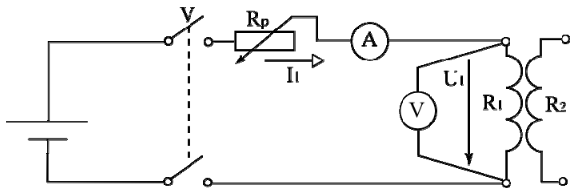

Fig. 2. The winding resistance measurement

Further, the no-load measurement (Fig. 3) was carried out.

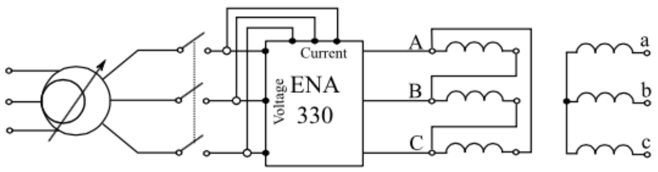

Fig. 3. The circuit diagram of the no-load measurement

The aim of the measurement is to determine the no-load loss P_0, the no-load current I_0 and the no-load power factor cos ϕ_0 at the rated voltage. The components of the equivalent circuit parallel branch are calculated using (7, 8, 9, 10) and the measured values. The mean measured values are presented in Table II, where I_{0z} and I_{0f} present no-load line-to-line and no-load phase currents, respectively.

TABLE II. THE EXAMPLE OF THE MEASURED VALUES OF THE NO-LOAD MEASUREMENT

U_{0f} (V)	246.7	214.4	187	162.6	106.2	78.3	29.1	13.2	3.47
I_{0z} (mA)	11.9	10.4	9.5	8.6	6.9	6.1	3.7	2.5	0.88
I_{0f} (mA)	6.8	6.0	5.4	5	4	3.5	2.1	1.3	0.51
P_0 (W)	3.7	2.9	2.3	1.8	0.9	0.5	0.1	0.02	0.00
Q_0 (var)	3.4	2.4	1.9	1.5	0.9	0.6	0.1	0.05	0.00
S_0 (VA)	5.1	3.8	3	2.4	1.2	0.8	0.2	0.04	0.01
cos ϕ (-)	0.7	0.7	0.7	0.7	0.7	0.6	0.4	0.3	0.22
ϕ (°)	42.2	39.7	39.4	40	44.6	49.3	64.3	71.7	77.15

The components of the equivalent circuit series branch are determined using short-circuit measurement (Fig. 4). The mean measured values of the short-circuit measurement are presented in Table III, where I_{kz} and I_{kf} present short-circuit line-to-line and short-circuit phase currents, respectively.

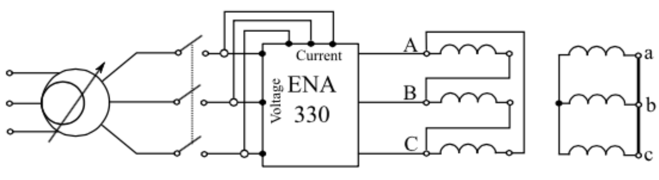

Fig. 4. The short-circuit measurement diagram

The equivalent circuit of the transformer with T-circuit shape is shown in Fig. 5. It is always drawn just for one phase, also in the case of multi-phase transformer, if the symmetry of the phases is assumed. Therefore, the voltages and the currents are always phase values. This fact is particularly important to take into account with the three-phase transformers, by which the line-to-line voltage is usually measured in the star connection and the line-to-line current in the delta connection.

TABLE III. THE EXAMPLE OF THE MEASURED VALUES OF THE SHORT-CIRCUIT MEASUREMENT

U_{kf} (V)	1.7	7.9	13.1	15.1	21.5	25.4	35.5	38.5
I_{kz} (mA)	8.4	39.2	63.3	74.1	105.4	124.7	174.2	189
I_{kf} (mA)	4.8	22.6	36.5	42.7	60.8	71.9	100.5	109.1
P_k (W)	0.02	0.5	1.4	1.9	3.9	5.4	10.6	12.6
Q_k (mvar)	2.3	41.2	101.8	135.6	262.6	355.7	697	807
S_k (VA)	0.02	0.2	0.4	0.6	1.3	1.8	3.5	4.2
cos ϕ_k (-)	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
ϕ (°)	5.3	4.3	4.1	4	3.8	3.7	3.7	3.6

The components of the equivalent circuit from Fig. 5 were arranged in the parallel and the series branches. The components, the voltages and the currents of the secondary side were converted to the primary side [1].

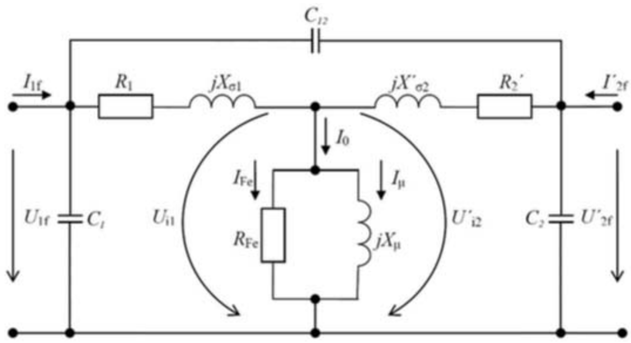

Fig. 5. The equivalent circuit

Capacitances C_1, C_2 a C_{12} were measured using RLC bridge, with the interconnected terminals of the windings (Fig. 6). The capacitances of the primary winding, between the secondary winding and the earth chassis and between the primary and secondary winding are C_1 = 98 nF, C_2 = 62 nF, C_{12} = 183 nF, respectively.

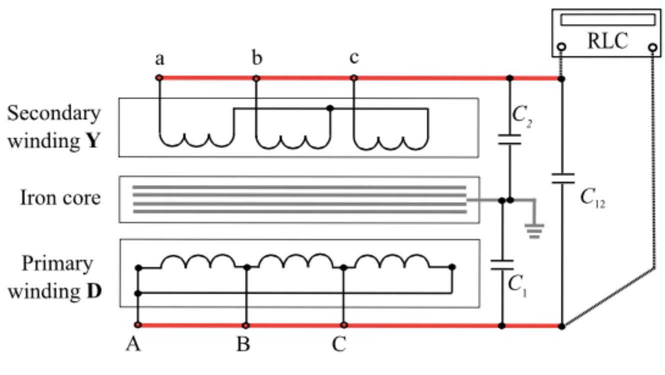

Fig. 6. Measurement of the parasitic capacitances

The short-circuit impedance was determined as:

$$Z_k = \frac{U_{kf}}{I_{kf}} = \frac{38.5}{109.1 \cdot 10^{-3}} = 352.8\,\Omega\,. \quad (2)$$

The leakage reactance was determined as:

$$X_\sigma = Z_k \cdot \sin\phi_k = 352.8 \cdot \left(\sqrt{1-0.9^2}\right) = 153.7\,\Omega\,, \quad (3)$$

and it could be divided between the primary and secondary winding as:

$$X_{\sigma 1} = X'_{\sigma 2} = \frac{X_\sigma}{2} = \frac{153.7}{2} = 76.89\,\Omega\,. \quad (4)$$

In view of the values of primary winding the resistances and of the current flowing through the parasitic capacitance C_1, they cannot be neglected. Therefore, to calculate the electromotive force, it is necessary to calculate the voltage drop across the primary branch of the equivalent circuit.

$$I_{C1} = -j \cdot \frac{U_1}{\frac{1}{2\cdot\pi\cdot f\cdot C_1}} = \frac{246.7}{\frac{1}{2\cdot\pi\cdot 50\cdot 183\cdot 10^{-9}}} = -j\cdot 14.7\,\text{mA}. \quad (5)$$

$$U_i = U_1 - \left(I_{1f} - I_{C1}\right)\cdot\left(R_1 + j\cdot X_{\sigma 1}\right). \quad (6)$$

$$U_i = 246.7 - \left(6.8\cdot 10^{-3}\angle -42.2 - (j\cdot 14.7\cdot 10^{-3})\right)\cdot(98.76 + j\cdot 76.89) = 245.5\angle 1.5\,\text{V}. \quad (7)$$

$$Y_0 = \frac{\left(I_{1f} - I_{C1}\right)}{U_i} = \frac{19.9\cdot 10^{-3}\angle -75}{245.5\angle 1.7} = 8.1\cdot 10^{-5}\angle -77\,s\,. \quad (8)$$

$$Y_0 = G + j\cdot B\cdot\mu\,. \quad (9)$$

$$R_{Fe} = \frac{1}{G} = \frac{1}{1.82\cdot 10^{-5}} = 54945\,\Omega\,. \quad (10)$$

$$X_\mu = \frac{1}{j\cdot B\cdot\mu} = \frac{1}{-7.9\cdot 10^{-5}} = 12658\,\Omega\,. \quad (11)$$

Where Y_0 is the magnetizing admittance, $j\cdot B\cdot\mu$ is an imaginary part of the magnetizing admittance and G is a real part of the magnetizing admittance.

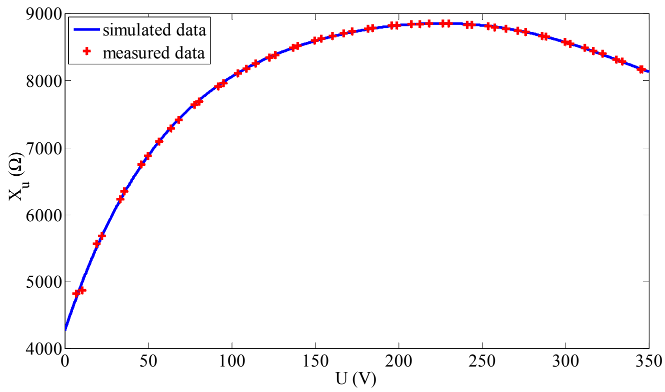

Fig. 7. Magnetizing reactance – voltage characteristic

Since the magnetizing reactance is not linear but varies according to the magnetization current, it is necessary to substitute it with a suitable polynomial (Fig. 7). The magnetizing reactance is not considered as a function of the magnetizing current, but as a function of the voltage in the model

$$X_\mu = f(U)\,. \quad (12)$$

The parameters of the function were determined empirically (13):

$$X_\mu = \left(4.7\cdot 10^{-9}\cdot U\right)^5 + \left(-5.4\cdot 10^{-6}\cdot U\right)^4 + \left(2.4\cdot 10^{-3}\cdot U\right)^3 + \left(-0.5\cdot U\right)^2 + \left(74.9\cdot U\right)^1 + 4.2\cdot 10^3\cdot U^0 \,. \quad (13)$$

It was identified from the measured values that even the resistance representing the iron loss is not linear but changes its value depending on the supply voltage (Fig. 8). The parameters of this polynomial was also established empirically (14).

$$R_{Fe} = \left(-2.2\cdot 10^{-7}\cdot U\right)^5 + \left(2.2\cdot 10^{-4}\cdot U\right)^4 + \left(-0.08\cdot U\right)^3 + \left(13.6\cdot U\right)^2 + \left(-922.8\cdot U\right)^1 + 5.5\cdot 10^4\cdot U^0 \,. \quad (14)$$

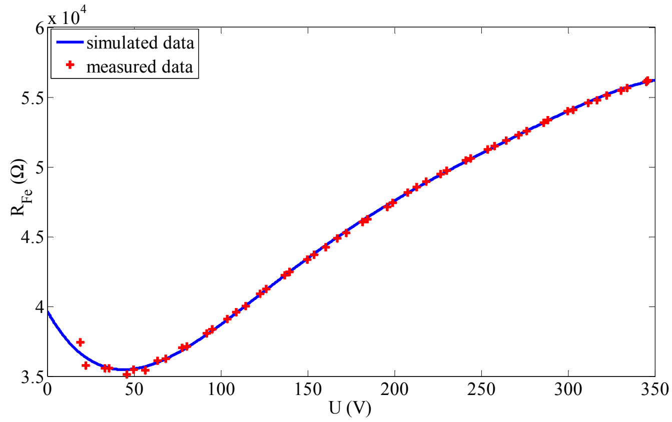

Fig. 8. Resistance representing iron losses – voltage characteristic

The simulation model in Matlab is created for this layout of the equivalent circuit (Fig. 9).

```
XX1=polyfit(U1,Xu1,5);
YY1=polyfit(U1,Rfe1,5);
ii=0;
for UU=0:0.1:350
   ii=ii+1;
   Xum=polyval(XX1,UU);
   Rfem=polyval(YY1,UU);
   A=R1+j*Xs1s;
   B=R2p+j*Xs2p;
   C=j*Xum*Rfem/(j*Xum+Rfem);
   D=Rz*Xc2p/(Rz+Xc2p);
   Ru=C*B/(B+C+D);
   Rv=C*D/(B+C+D);
   Rw=B*D/(B+C+D);
   E=A+Ru;
   F=Xc12+Rw;
   G=E*F/(E+F);
   H=G+Rv;
Z=Xc1*H/(Xc1+H);
   II(ii)=abs(UU/Z);
   UUu(ii)=UU;
   Xumm(ii)=Xum;
   Rfemm(ii)=Rfem;
end
```

Fig. 9. The part of the simulation model in Matlab

An error in determining the components of the equivalent circuit in view of the magnitude of the capacitances C_2 and C_{12}

is so small that they can be neglected. Reactance of these capacitances converted to the primary side are so large that they form only a small part of measured current I_0, thus neglecting these currents do not significantly affect the determination of the R_{Fe} a X_u (Fig. 5).

The result of the simulation is the comparison of the no-load (Fig. 10), short-circuit measurement (Fig. 11) and measurement of the loaded transformer (Fig. 12), with the results obtained from the simulation. Simulation of the no-load measurements were processed as follows. As the load resistance the value of R_{load} = 10000000 is chosen, which can be considered as sufficiently high resistance. When simulating short-circuit measurement, the value of the load resistance R = 0 is chosen. Then, the values of the voltage from 0 V to 350 V in case of no-load and to 50 V for the short-circuit are increased with step 0.1 V and the values of the current are determined.

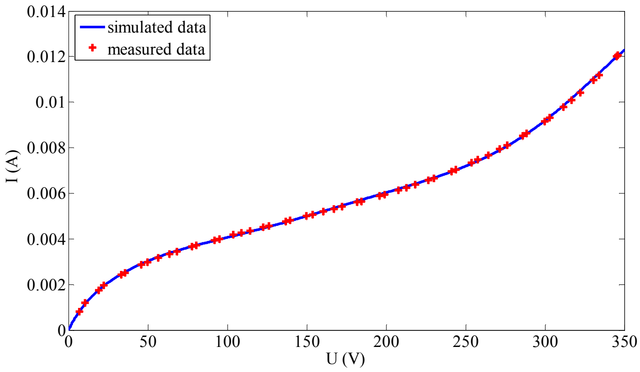

Fig. 10. The comparison of the obtained results for no-load measurement

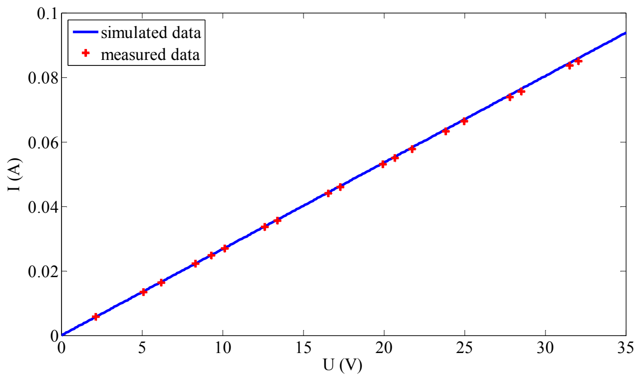

Fig. 11. The comparison of the obtained results for short-circuit measurement

Measurement of the loaded transformer was simulated in the similar way. In this case, the supply voltage was constant U_1 = 220 V, while the load resistance R_{load} was changing. The measurement was carried out for five different values of resistance and they are 1, 2, 3, 4, and 5 Ω. (Tab. IV).

TABLE IV. THE EXAMPLE OF THE MEASURED VALUES OF THE LOADED TRANSFORMER

R_{load} (Ω)	5	4	3	2	1
I_1 (mA)	11.2	12.4	14.8	19.1	33.4

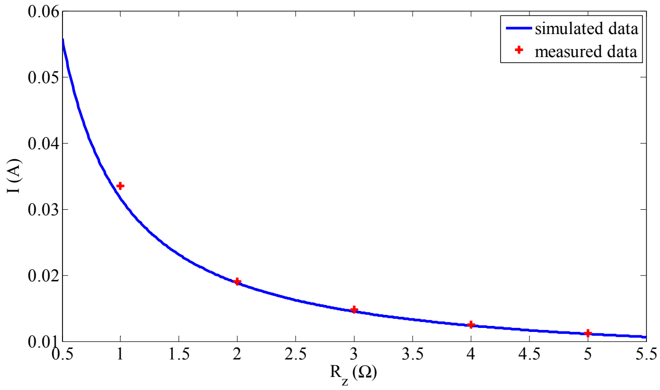

Fig. 12. The comparison of the obtained results for loaded transformer

III. CONCLUSION

The transformer data necessary to determine the components of the equivalent circuit were obtained from the measurements. Then, the model was created, which in comparison with the standardly used models of the transformer was complemented with the parasitic capacitances. Simulation results of the mathematical model in Matlab were compared with the results obtained from the measurement and show a good coincidence. This model will be used for further investigation of the transient phenomena, the resonance and the operation of the transformer with the inharmonious power supply.

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